silicate to carbonate, and then back again to silicate, the only limit to the process would be the satisfying of the mutual affinities of the silica and the basic oxyds present.

III. "On the Comparison of Transcendents, with certain applications to the Theory of Definite Integrals." By GEORGE BOOLE, Esq., Professor of Mathematics in Queen's College, Cork. Communicated by Professor W. F. DONKIN, F.R.S. Received March 16, 1857.

(Abstract.)

The following objects are contemplated in this paper:—

1st. The demonstration of a fundamental theorem for the summation of integrals whose limits are determined by the roots of an algebraic equation.

2ndly. The application of that theorem to the comparison of algebraical transcendents.

3rdly. Its application to the comparison of functional transcendents, *i. e.* of transcendents in the differential expression of which an arbitrary functional sign is involved.

4thly. Certain extensions of the theory of definite integrals both single and multiple, founded upon the results of the application last mentioned.

In the expression of the fundamental theorem for the summation of integrals, the author introduces a symbol, Θ, similar in its definition to the symbol employed by Cauchy in the Calculus of Residues, but involving an additional element. The interpretation of this symbol is not arbitrary, but is suggested by the results of the investigation by which the theorem of summation is obtained. All the general theorems demonstrated in the memoir either involve this symbol in their expression, or are immediate consequences of theorems into the expression of which it enters.

The author directly applies his theorem of summation both to the solution of particular problems in the comparison of the algebraical transcendants, and to the deduction of general theorems. Of the

latter the most interesting, but not the most general, is a finite expression for the value of the sum

$$\Sigma\int\phi\psi^{\frac{m}{n}}dx,$$

where ϕ and ψ denote any rational functions of x; the equation by which the limits of the integrals are determined being of the form $\psi^{\frac{m}{n}}=\chi$, in which χ is also a rational function of x.

The forms of ϕ, ψ, and χ are quite unrestricted, except by the condition of rationality. Previous known theorems of the same class, such as Abel's, suppose ψ a polynomial and specify the form of ϕ. In the author's result, the rational functions ϕ, ψ, and χ are not decomposed. In a subsequent part of the paper, after investigating a general theorem applicable to the summation of all transcendents which are irrational from containing under the sign of integration any function which can be expressed as a root of an equation whose coefficients are rational functions of x, he explains, by means of it, the cause of the peculiarity above noticed.

In the section on functional transcendents, a remarkable case presents itself in which the several integrals under the sign of summation, Σ, close up, if the expression may be allowed, into a single integral taken between the limits of negative and positive infinity. The result is an exceedingly general theorem of definite integration, by means of which it is demonstrated, that the evaluation of any definite integral of the form

$$\int_{-\infty}^{\infty}\phi(x)f\left(x-\frac{a_1}{x-\lambda_1}-\frac{a_2}{x-\lambda_2}\cdot\cdot-\frac{a_n}{x-\lambda_n}\right)dx,$$

in which $\phi\,(x)$ is a rational function of x, and in which $a_1\ a_2 \ldots a_n$ are positive, and $\lambda_1, \lambda_2 \ldots \lambda_n$ are real, the number of those constants being immaterial, may be reduced to the evaluation of a definite integral of the form

$$\int_{-\infty}^{\infty}\psi(v)f(v)dv,$$

in which $\psi(v)$ is a rational function of v of the same order of complexity as the function $\phi(x)$. Two limited cases of this theorem are referred to as already known,—one due to Cauchy, the other published by the author some years ago.

The remainder of the paper is occupied with applications of the

above general theorem of definite integration. Of the Notes by which the paper is accompanied, the first discusses the connexion between the author's symbol and Cauchy's, and contains two theorems, one exhibiting the general solution of linear differential equations with constant coefficients, the other the general integral of rational fractions. Both these theorems involve in their expression the symbol Θ. The second Note is devoted to the interpretation of some theorems for the evaluation of multiple integrals, investigated in the closing section of the paper.

May 14, 1857.

General SABINE, R.A., Treas. and V.P., in the Chair.

The following communications were read :—

I. "On the Organization of the Brachiopoda." By ALBANY HANCOCK, Esq. Communicated by T. H. HUXLEY, Esq., F.R.S. Received April 24, 1857.

(Abstract.)

In the present memoir the author states at length, and fully illustrates by figures, the conclusions to which he has been led by a long series of researches into the anatomy of the Brachiopoda; investigations which have been conducted with a special reference to the discrepant opinions maintained by Prof. Owen and the older writers on the one hand, and by Prof. Huxley and himself on the other. Some of the points in dispute have already been discussed in a paper read before the British Association at Cheltenham, and in the present memoir the author not merely reiterates the statements which he then made, but gives a detailed account of the whole organization of the Brachiopoda based upon his dissections of the following species :—*Waldheimia australis*, *W. Cranium*, *Terebratulina caput-serpentis*, *Rhynchonella psittacea*, *Lingula anatina*, and another species of *Lingula*.

The Brachiopoda are divisible into two groups, according as the valves of their shells are articulated or not. *Waldheimia* is the type of the former group, *Lingula* of the latter.